

# ABSTRACTS

## MAA SEAWAY SPRING MEETING

### HAMILTON COLLEGE, 2012

#### Friday night speaker

**David Bressoud, Macalester College**

#### *Stories from the Development of Real Analysis*

Analysis is what happened to calculus in the 19th century as mathematicians discovered that their intuition of how to apply calculus was failing them, especially as their repertoire of infinite series expanded. The conceptual difficulties that they encountered are precisely where we should expect our own students to have trouble. Understanding how these controversies were resolved illuminates many of the definitions, axioms, and theorems that baffle our students. This talk will focus on the early 19th century and consider two questions: What do we mean by convergence of a series of functions and when, for the purposes of calculus, can we treat an infinite sum of functions as if it were a finite sum? And, how did our modern understanding of the Fundamental Theorem of Calculus arise, and what does it really say?

#### Saturday morning speakers

**Bill Dunham, Muhlenberg College**

#### *Two (more) Morsels from Euler*

Euler's 2007 tercentenary generated a number of talks about his celebrated mathematical triumphs. Here we examine a pair of lesser-known theorems where his genius was on full display. In the first, we consider Euler's response to the challenge of finding four different whole numbers, the sum of any pair of which is a perfect square. With characteristic ingenuity, he came up with the fearsome foursome of 18530, 38114, 45986, and 65570. We'll look over his

shoulder to see how he did it. Moving from number theory to analysis, we examine his summation of the series of reciprocals of squares – i.e.,  $1 + 1/4 + 1/9 + 1/16 + \dots$  – as presented in his 1755 text on differential calculus. The amazing thing about this derivation is that it used l'Hospital's rule ... not once, nor twice, but thrice! These two results, which require only elementary mathematics, are reminders of why Euler should be celebrated on his 300th birthday and always.

## **Robin Lock, St. Lawrence University**

### *Bootstrapping: Let your data be your guide*

The concept of bootstrapping was introduced by Brad Efron in the late 1970's as a computer-intensive simulation method to gain information about the distribution of a sample statistic using only the data in the sample itself. Advances in computer technology have made this technique increasingly accessible and useful, but common initial reactions are that it is either "magic" or "cheating" (analogous to elevating yourself by pulling hard enough on your boot laces). This talk will attempt to de-mystify the procedure and show how/why it works (and when it doesn't). We also discuss how this approach can be made accessible to students with minimal background as a way to introduce important ideas of statistical inference at the early stages of a course.

## **Gehman Lecture: Col. Steve Horton, United States Military Academy**

### *Optimal Generation and Packing of Steiner Trees in a Rectilinear Grid*

In this research, we seek to find an algorithm for an optimal generation and packing of Steiner trees in a rectilinear grid. This problem arises in computer chip design where a set of terminals on a prescribed set of components needs to be connected using a wiring "tree". Such a set of terminals is called a net. These trees are so-called Steiner trees as we are permitted to add new nodes - Steiner nodes - to reduce their total length. In general, many sets of terminals need to be connected using different, disjoint Steiner trees. A chip we are wiring has limited space for these trees and all of these must be packed onto the chip without interfering with each other.

## Saturday afternoon speakers

**Christopher Baltus**, SUNY Oswego

*Brook Taylor in Perspective: Perspective Drawing as a Central Collineation*

Although practical methods for perspective drawing developed in fifteenth century Italy, the mathematical treatment emerged slowly. Major steps in that direction were G. J. 'sGravesande's *Essai de Perspective* (1711) and Brook Taylor's *Linear Perspective: or a new method . . .* (1715). Their methods rotated the "picture plane" onto the "ground plane," so that images could be constructed by what we now recognize as a central collineation. The geometry is elementary and very pretty.

**Joel Dreibelbis**, Rochester Institute of Technology

*Dynamics of Linear Maps*

Bezout's theorem is a classic result regarding the number of common points between two curves (when there are finitely common points, the maximum number of common points is bounded by the degrees of the curves). For a self-map  $\Phi$  from  $S$  to  $S$  (one may take  $S$  to be the set of ordered pairs of real numbers) and a starting point  $q$  in  $S$ , the orbit set of  $q$  under  $\Phi$  is the set of points:  $\{q, \Phi(q), \Phi(\Phi(q)), \dots\}$ . An interesting analog of Bezout's theorem is to consider the maximum number of common points between an orbit set and a curve. Results will be discussed in the case where  $\Phi$  is a linear map which culminates in a uniform bound for the dynamical Mordell-Lang conjecture for linear maps.

**David Farnsworth**, Rochester Institute of Technology

*A Statistical Test for Mutual Exclusivity*

A statistical test for two events being mutually exclusive in an environment of misclassifications is presented. The test can be used to test whether a set is a subset of another set. Examples of medical testing with possible misdiagnoses are given.

**Keary Howard, SUNY Fredonia**

*Mathematics Cognition and Misconceptions in Introductory College Mathematics Courses*

This session is highlighted by the presentation of research results from secondary school mathematics teachers completing their capstone Masters projects. Topics and presenters include:

Zachary Teetsel, SUNY Fredonia

*What the Words Say: A Study of Students' Efficacy in Mathematical Vocabulary*

Michael Humber, SUNY Fredonia

*Making Sense without Number Sense: A Study on Student Self-Perception of Algebra Skills versus Number Sense.*

Mimi Gillick, SUNY Fredonia

*Anxiety in Action: Math Anxiety in College Freshmen and the Exhibited Differences among Math Majors and Non-Math Majors*

Michelle Terranova, SUNY Fredonia

*Name that Function: An Examination of Function Recognition between Mathematics Majors and Non-Majors*

**Chulmin Kim, Rochester Institute of Technology**

*Which works better to predict the 2012 NCAA bracket? RPI or BPI? Or Sagarin's?*

The NCAA Basketball Championship is a single-elimination tournament held each spring (mostly in March so that it is informally known as March Madness), featuring 68 college basketball teams, to determine the national championship. The field of 68 includes the champions from 31 Division I conferences (automatic bids) and the other 37 selected at-large berths by the selection committee. The brackets and seeds are also released by the committee. A number of factors including winning ratio, schedule strength, and the recent results are used to place the teams from the first to the last. The Rating Percentage Index (RPI), one of the most prominent power rating systems, is often considered the biggest decision aid for the committee. RPI has been popular due to its simplicity however it doesn't account for the actual scores. The College Basketball Power Index (BPI) is recently introduced by ESPN. It is not only based on the factors in

the RPI, but also the scoring margins, pace of games, and more. The RPI, the BPI, and the Sagarin's (another famous rating system) are evaluated by various statistical ways for their abilities to predict the 2012 NCAA bracket.

**Yozo Mikata**, Bechtel, Schenectady, NY

*Deformation of CNT (Carbon Nanotube) by Molecular Mechanics*

In this talk, we will discuss the tensile rigidity (EA) and Poisson ratio of carbon nanotube using molecular mechanics. In addition, we will also discuss the stress-strain curve of CNT, which is obtained up to a relatively large strain level (15%). The solution method is primarily numerical, and the calculations are done by Mathematica. Comparison will be made with similar results obtained by experimental, computational and analytical methods in the existing literature.

**Olympia Nicodemi & Patrick Rault**, SUNY Geneseo

*Sharing our co-teaching experiences*

P. Rault and O. Nicodemi are team-teaching inquiry-based Abstract Algebra this semester. Last semester we aligned our sections of lecture-based linear algebra closely and consulted continually. We would like to share our experience and the questions it raised about teaching and especially about teaching using inquiry based learning.

**Sam Northshield**, SUNY Plattsburgh

*Ellipses and results of Marden and Cardano*

For complex  $z$ ,  $a$ ,  $b$ , the linear map  $L(z) = az + b\bar{z}$  takes line segments to line segments, midpoints to midpoints and circles to ellipses. This leads to a short new proof of 'Marden's theorem' (that Prof. Dan Kalman called "the most marvelous theorem of mathematics" which relates the roots of a cubic polynomial with the roots of its derivative). It also allows a geometric understanding of Cardano's formula for finding the roots of a cubic polynomial.

**Joseph Petrillo**, Alfred University

*The Alfred University Calculus Initiative*

The Alfred University Calculus Initiative (AUCI) is a multi-faceted project that combines a new and distinctive curriculum with classroom transformation, video lessons, online homework, and web-based implementation into a comprehensive calculus experience. The goal of the AUCI is to increase understanding and success in calculus and precalculus while maintaining the level of rigor and breadth required for post-calculus courses. This project is being informed by current research and trends in STEM education, which include engaging students with visual and online technology, creating an active learning environment in the classroom, and incorporating meaningful applications. In this talk, we will discuss the motivation and content of the new curriculum and give convincing data that supports its effectiveness.

**Gabriel Prajitura**, SUNY Brockport

*Approximating by averages*

We will show how certain rational means can be used to approximate irrational roots.

**Victor Protsak**, SUNY Oswego

*On the shape of a dinner napkin*

What is the shape of a dinner napkin pinned at the corners? I do not know a precise answer to that question, which was raised on mathoverflow by Joseph O'Rourke, but an easier problem involving a napkin suspended at one corner has a surprising twist. It leads to a pleasant computation involving elementary geometry and basic calculus. Bring your own silk handkerchief for some hands-on exploration!

**Paul Seeburger**, Monroe Community College

*Verifying Surface Intersection Curves Visually*

In multivariable calculus we ask students to determine the parametric equations for the curve of intersection of two surfaces. This includes finding the intersection of two planes (a three-dimensional line), and the intersection of a pair of surfaces including spheres, paraboloids, hyperboloids, and planes. Using a freely available online multivariable calculus applet called CalcPlot3D, a pair of surfaces can be graphed along with the intersection curve (entered parametrically), and the intersection curve can be verified visually. 3D glasses can be used for a real 3D perspective! CalcPlot3D is part of an NSF-funded grant project called Dynamic Visualization Tools for Multivariable Calculus (DUE- CCLI #0736968). See <http://web.monroecc.edu/calcNSF/>. This lesson includes an example to demonstrate in class and a worksheet for students to complete as homework. Students are asked to use the CalcPlot3D applet to graph the surfaces in their problems along with the space curves they obtain to represent the surface intersections. The applet allows the graphs to be printed and includes a date/time stamp and the name of the computer used for the exploration.

**Stan Seltzer**, Ithaca College

### *The Top-ten list*

What are the ten most important and/or interesting mathematical things (results, trivia, etc.) that your majors **won't** see in your curriculum? And what can you do about it? In this talk I'll share my list and tips for using it and other top ten lists.

**Robert Sulman**, SUNY Oneonta

### *Orbits under polynomials modulo $n$ that coincide with subgroups of the units of $\mathbb{Z}/n\mathbb{Z}$ modulo $n$*

Given the group  $\mathbf{G} = ((\mathbb{Z}/n\mathbb{Z})^*, \cdot_n)$  and a polynomial  $f: \mathbf{G} \rightarrow \mathbf{G}$ , one can ask if there is a connection between the orbit of 1 (modulo  $n$ ) and the algebraic structure of  $\mathbf{G}$ . When  $f$  is injective (which can easily be arranged when  $n$  is a power of 2), the orbit of 1 and of any other element of  $\mathbf{G}$  will be a cycle, since  $f$  would be an element of the symmetric group,  $S_{\varphi(n)}$ . Often, such an orbit of 1 is a subgroup of  $\mathbf{G}$ . This outcome is certainly the case when this cycle has length  $\varphi(n)$ . When  $f$  is a product of more than one cycle, the orbit of 1 may coincide with a (proper) subgroup  $\mathbf{H}$  of  $\mathbf{G}$ . In this case there is a natural yet varied correspondence between the cosets of  $\mathbf{H}$  and the cycles of  $f$ . Furthermore, the orbit of 1 under  $f^2$  (where the exponent refers to composition) is seen to be a cyclic subgroup of  $\mathbf{G}$ . We also examine cases where  $f(\mathbf{H})$  or  $f^{-1}(\mathbf{H})$  are subgroups of  $\mathbf{G}$ . Finally, given

two polynomials  $f$  and  $g \pmod{n}$ , we consider orbits of composites (words in  $f$  and  $g$ ), and their relationship to  $\mathbf{G}$ .

**Marshall Whittlesey**, California State University San Marcos

*Teaching spherical geometry to undergraduates*

A century ago, spherical geometry (the study of geometric objects on the surface of a 3-dimensional ball) was a standard part of the mathematics curriculum in high schools and colleges. Its applications were needed by many people: anyone who wanted to navigate on the surface of the earth by using the stars needed to know something about the subject. However, in the decades since the 1940s, it has slowly disappeared from the curriculum for most students, and today most mathematicians only learn about it as a short topic in geometry survey courses. In this talk we survey some of the standard theorems of spherical geometry and compare them to those of plane geometry. We also will discuss some of the interesting applications of spherical geometry in astronomy, crystallography and geodesy. We suggest spherical geometry as a good subject for future high school teachers to learn, but also think more mathematicians should be generally aware of its theorems and applications.

**Elizabeth Wilcox**, Colgate University

*I Fold: Origami as a Technique*

Have you heard? Given an arbitrary angle you could spend all day working with your compass and (unmarked) straightedge to no avail -- unless you happened to be given one of a few specific angles, you could not divide your angle into three equal sub-angles. So what do mathematicians do when their hopes of solving a problem have been dashed? We either change the problem, or change the tools we use to solve that problem! When it comes to trisecting angles, why stick to a compass and unmarked straightedge? What other tools do we have that might allow us to achieve trisection? Obviously a protractor will do the job, if you are very careful with your measurements. So the real question is: What is the most basic tool available that allows us to trisect any given angle? It turns out that you need only your hands, paper, and a little dexterity to solve this problem... no other tools are necessary. Let me show you how origami is the slickest tool in the geometer's toolbox... for this problem, at least!

**Xiao Xiao**, Utica College



### *The Frobenius Problem*

Let  $a, b, \dots, c$  be positive integers with the greatest common divisor equal to 1. The Frobenius problem studies the largest non-negative integer  $g(a, b, \dots, c)$  that cannot be expressed as a non-negative linear combination of  $a, b, \dots, c$ . In this talk, I will recall some old and new results of solving the Frobenius problem for  $g(a, b)$  and  $g(a, b, c)$ . If time permits, I will also describe how a closed formula of  $g(a, b, c)$  can be used to solve problems in the classification of  $F$ -crystals. This talk is accessible to undergraduates.